Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Influence of Finite Slot Size on Boundary Layer with Suction or Injection"

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THE author should be congratulated on his timely and interesting Note.[1] The effects of injection described by the author are relevant to any flow and subsequently to the heat transfer analysis of porous-wall materials for combustor linear cooling[2] or film cooled surfaces[3] of high pressure axial turbine blades and vanes at low freestream Mach numbers. Aerodynamic means to minimize the losses associated with film cooling are mostly empirical in nature and their relative merits and applicability must be established (usually through experiments) for each unique combination of the blade surface velocity distribution and the pressure drop across the coolant holes. The application of the previously developed inviscid-viscous interaction numerical method to injection/suction through slots described in this Note[1] appears to be another significant step in that direction.

Since the solution of the Navier-Stokes equations relieves restrictions regarding primary flow direction and separation regions, as well as providing sufficient generality to deal with variable injection rates, angles, and other flow (and heat transfer) parameters, it would be more appropriate to study the crucial interaction between the injected flow and the mainstream by extending the Navier-Stokes model[1] of the slot flow to include some part of the computational domain upstream, downstream, and above the top of the slot. The elliptic stream-function-vorticity formulation[1] could be coupled with a parabolic marching scheme downstream of the recirculation region, a few slot length beyond the trailing edge of the slot. The adequacy of such calculations using a similar formulation[5] has been established[3] at DDA for the injection through a slot.

Two different sets of Reynolds numbers are required in the analysis[4]: 1) a Reynolds number for the mathematical model for the shear layer, and 2) a Reynolds number based on the flow conditions within the slot. The FTCS (forward-time center-space) numerical technique used by the author[1,4] has a comparatively more restrictive stability requirement for convergence than some of the other numerical methods available[6] for the stream function-vorticity formulation, and the method is likely to encounter problems associated with convergence at higher blowing rates.

In addition, the author should elaborate on the treatment of the singularity at the sharp corners of the leading and trailing edges of the slot (to detemrine the dividing streamline), as this could play a dominant role in the phenomenon of separation and reattachments which would consequently influence the inflow of hot gases into the slot for film cooling applications.

Also, for the two slot sizes considered in the Note, what were the ratios of L_s/(slot height) used in the analysis such that the constant vertical velocity boundary condition did not affect the slot-mainstream interface and the type of velocity distribution (parabolic, power-law, etc.) assumed upstream of the slot for the solution of the compressible boundary-layer equations?

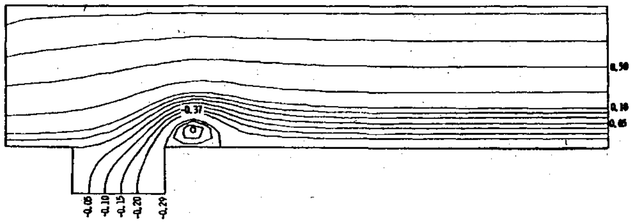

Fig. 1 Streamlines for normal injection, $M = 0.25$.

It is an established fact that the normal injection of fluid into a freestream has a destabilizing effect on the flowfield, and the probability of the formation of a small separation bubble downstream of the trailing-edge surface of the slot becomes larger with increasing blowing ratios. This is illustrated in Fig. 1 based on our calculations with normal injection through a slot at a blowing ratio, $M = 0.25$, for subsonic flow. Could the author illustrate the distribution of the x component of velocity at the trailing edge and slightly downstream of the slot for his supersonic flow investigation to clarify the disturbed nature of the shear layer in the vicinity of the slot?

In Fig. 3a of Ref. 1, for the slot with normal injection ($L_s = 0.008\ L_{BL}$), there is a significant convergence of the streamlines at $y \sim 0.2$. Does this imply a zone of recirculation on the two faces of the slot or is it due to the specification of a constant vertical velocity distribution at the bottom of the slot?

The relative performance of the various slots examined pertains to a specified pressure drop across the mainstream and the bottom of the slot and therefore the velocity distributions are not universally applicable. The primary benefit of this analysis lies in its potential for specifying the interaction between the flow within the slot and the mainstream; and if the boundary layer restriction in the viscous region above the slot is replaced by a Navier-Stokes formulation, it will be possible to analyze the flowfield with much larger velocity ratios (v/U_∞) than have been reported.

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Reply by Author to A. R. Wadia

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I WISH to thank A. R. Wadia for his interest and remarks concerning my Note.[1] In general, I am in agreement with the ideas expressed in his Comment, but I should like to add the following discussion and to answer the questions posed.

The definition of the Navier-Stokes region used in Ref. 1 is, as Wadia points out, restrictive in that it presupposes that flow reversal does not occur outside of the cavity. If this is true, then addition of the interaction with the third layer—the potential outer flow—is usually sufficient to account for the upstream propagation of disturbances through the subsonic portion of the boundary layer as well as to calculate the correct downstream behavior of the disturbed layer. Such a method was described in Ref. 2 and its use illustrated in Refs. 2 and 3. For the relatively low injection rates into the supersonic boundary layer in Ref. 1 ($v/U_\infty = 0.02$ for the outer flow of $M_\infty = 2.25$) the two-layer approach, as well as the simple choice of the rectangular Navier-Stokes region, were found appropriate for the purposes of that paper. The disturbed nature of the boundary layer downstream of the slot is evident nevertheless in the results obtained in Ref. 1. The qualitatively correct behavior of reduced shear downstream of the injection slot, and increased shear downstream of the suction slot, was reproduced. The computed values of the u velocity component is one grid interval above the horizontal wall (which is linked to $\partial u/\partial y$ at the wall) normalized by the corresponding value of this parameter upstream of the slot are persented in Table 1 for three x steps, starting at the downstream corner, and are intended for qualitative comparison only.

It was shown in the Comment that for a subsonic flow with a relatively high rate of injection ($v/U_\infty = 0.25$) a separation bubble can form downstream of the injection slot. To treat such a flowfield adequately using the interactive method of Refs. 1-4, the Navier-Stokes domain should be extended downstream (and perhaps upstream) of the slot to include at least all the regions of flow reversal, as Wadia recognized. If the incompressible equations are to be retained in the recirculation region and the outer flow is supersonic or highly subsonic so that the incompressibility assumption fails in a significant part of the boundary layer, then the matching region may no longer be specified a priori using a simple, rectangular geometry. Instead, curved matching boundaries, which can evolve as the solution proceeds, are necessary. Such a self-adaptive approach to the construction of the matching region is, at present, beyond the capability of the model, but is a natural subject for further work in extending the method.

The vorticity at the two convex corners of the slot was determined along the vertical walls for Navier-Stokes region

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Table 1 Normalized u-velocity component downwind of the slot

Mode	Suction					
Slot width	Narrow slot ($L_s = 0.004\,L_{BL}$)			Wide slot ($L_s = 0.008\,L_{BL}$)		
Distance from corner	C	Δx	$2\Delta x$	C	Δx	$2\Delta x$
$\left.\frac{u}{u_0}\right\vert_{\Delta y}$	1.98	1.54	1.55	2.84	1.75	1.90
Mode	**Injection**					
Slot width	Narrow slot ($L_s = 0.004\,L_{BL}$)			Wide slot ($L_s = 0.008\,L_{BL}$)		
Distance from corner	C	Δx	$2\Delta x$	C	Δx	$2\Delta x$
$\left.\frac{u}{u_0}\right\vert_{\Delta y}$	1.14	0.38	0.79	0.28	0.23	0.37

and along the horizontal wall for the boundary-layer computations.[4] This discontinuous wall value treatment[5] follows naturally from the definition of the two computational regions and is, in effect, the same as the method used by Wadia and Booth.[6] As the computational step size decreases, the corner becomes "sharper" numerically and the solution may be locally affected. For similar problems, it was found[3] that Δx variations of approximately one order of magnitude (while Δy was held constant) affected the stagnation point location by a fraction of Δy.

Both the centered and upwind differencing were tried for the convective terms in the numerical model. Because the cavity Reynolds numbers were low [0(10)] the results generated by both schemes were virtually identical. No convergence problems were encountered with either method in any of the cases examined. The primary constraint on applicability of the present approach is the selection of the matching boundaries.

The remaining details are now addressed. No closed recirculation regions were seen within the slot for the cases with injection (although separation beneath the upstream corner did appear for the narrow slot in Fig. 3a of Ref. 1).

The depth of the slot was 1.25 L_s and 2.5 L_s, respectively, for the wide and the narrow configurations. An initial boundary layer profile of the Blasius form was specified 25 narrow slot lengths ahead of the interaction.

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